MATH 245 F20, Exam 1 Questions
(60 minutes, open book, open notes)

1. Let b, c be odd integers. Without using theorems, prove that $b(c-2)$ is odd.
2. Prove or disprove: For all propositions p, q, the proposition $(p \uparrow q) \downarrow (p \leftrightarrow q)$ is a contradiction.
3. Let p, q, r, s be propositions. Prove that $p \vee q, q \wedge r, p \to s \vdash q \vee s$.
4. Prove the following without truth tables: For any propositions p, q, r, s, we have $p \to q, q \to r, r \to s \vdash p \to s$.
5. Let $x \in \mathbb{R}$. Prove that if x^2 is irrational, then x is irrational.
6. Fix our domain to be $\mathbb{Z}$ for all variables. Simplify the following proposition as much as possible (where nothing is negated): $\neg\ \forall x\ \forall y\ \exists z\ (x < y) \to (x < z \leq y)$.
7. Prove or disprove this proposition: $\forall x \in \mathbb{Z},\ \exists y \in \mathbb{Z},\ (x \neq y) \wedge (y|x)$.